Warm-ups:

① Graph $(x-2)^2+(z+1)^2=16$, $y>-4$

② Graph $(z+1)^2=16$ in $\mathbb{R}^3$.

③ Graph $x^2+y^2+z^2=25$, $y=-3$

④ Graph $x^2+y^2+z^2=144$, $y<0$.

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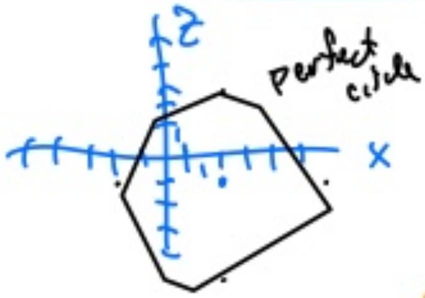
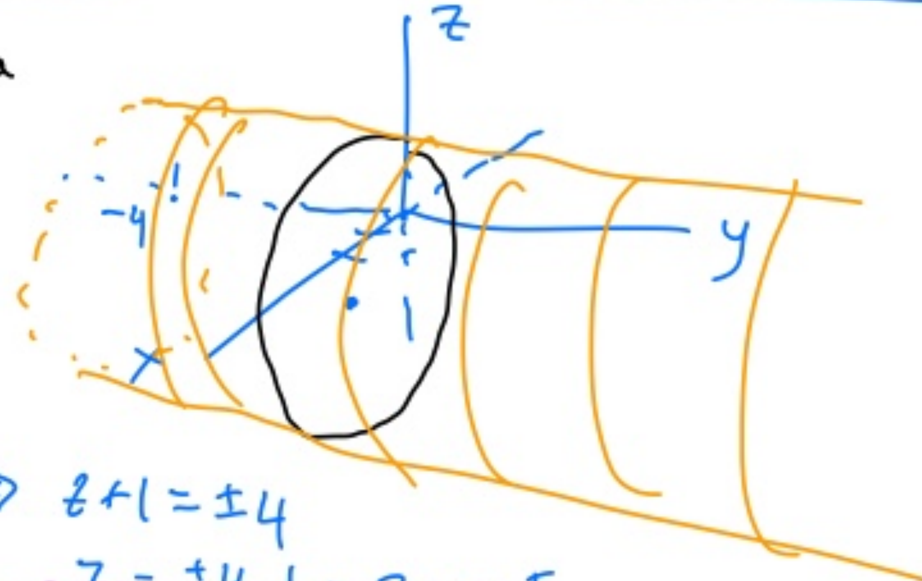
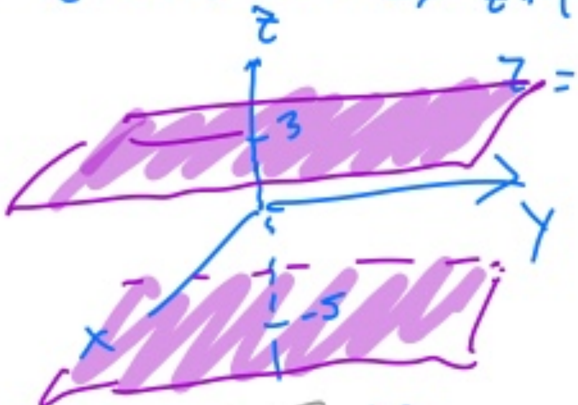
①



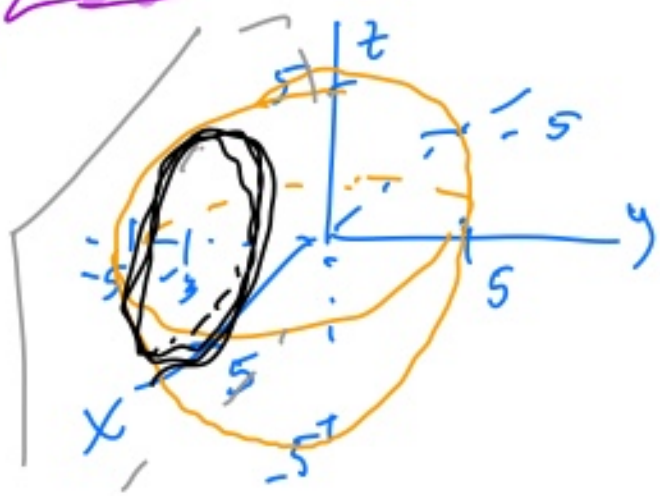


② $(z+1)^2=16 \Rightarrow z+1=\pm 4$

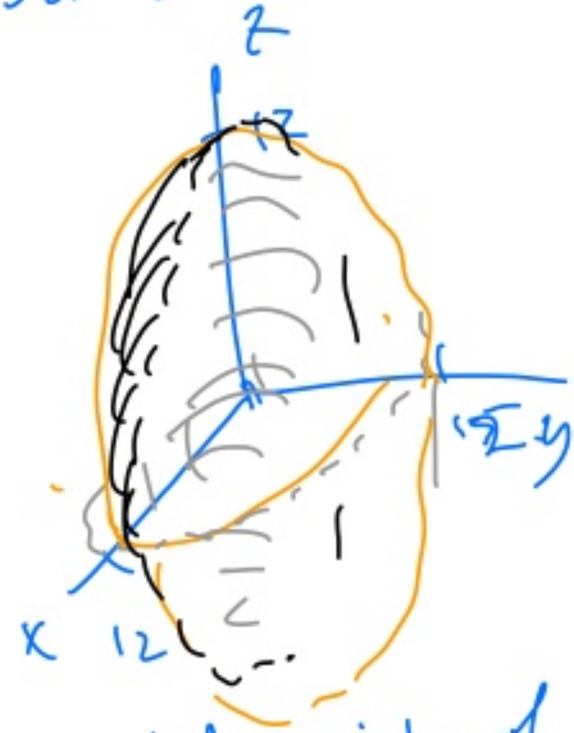
$z=\pm 4-1=3$ or -5



③

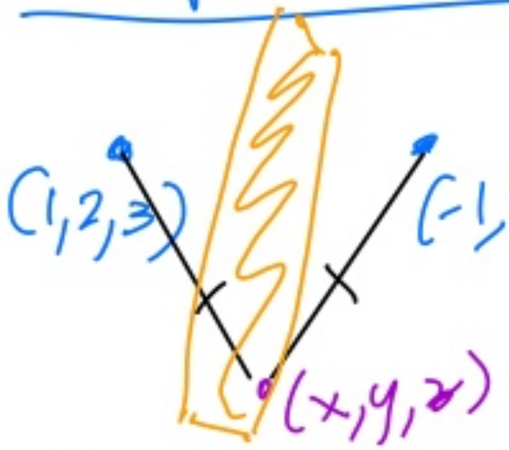


④



left hemisphere of radius 12, centered at (0,0,0)

⑤ Give the equation(s) for the set of points equidistant from $(1,2,3)$ & from $(-1,4,8)$.



literally

dist$((x,y,z) \to (1,2,3))$ = dist$((x,y,z) \to (-1,4,8))$

$$\sqrt{(x-1)^2+(y-2)^2+(z-3)^2} = \sqrt{(x+1)^2+(y-4)^2+(z-8)^2}$$

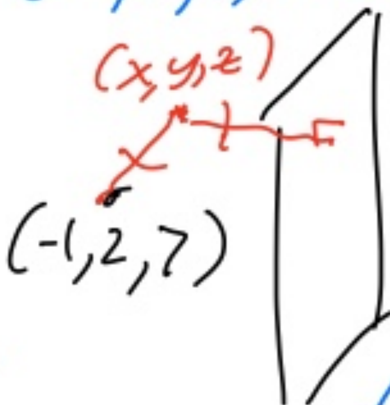
$$\Longleftrightarrow (x-1)^2+(y-2)^2+(z-3)^2 = (x+1)^2+(y-4)^2+(z-8)^2$$

$$\Longleftrightarrow \cancel{x^2}-2x+1+\cancel{y^2}-4y+4+\cancel{z^2}-6z+9 = \cancel{x^2}+2x+1+\cancel{y^2}-8y+16 + \cancel{z^2}-16z+64$$

$$\Longleftrightarrow -4x+4y+10z = 67$$ linear eqn in $\mathbb{R}^3$.

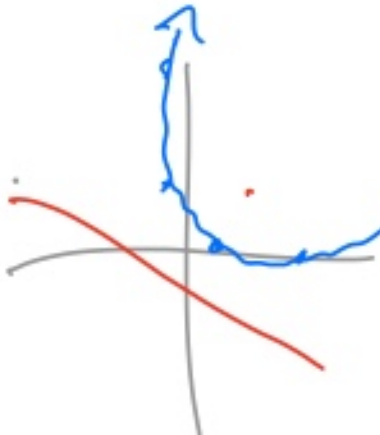
⑥ Find eqn of the set of pts equidistant from $(-1,2,7)$ and the xz plane.

$y=0$.



$$\sqrt{(x+1)^2+(y-2)^2+(z-7)^2} = |y|$$

(dist from (x,y,z) to $(-1,2,7)$) = (dist from (x,y,z) to xz plane)



$$\Longleftrightarrow (x+1)^2+(y-2)^2+(z-7)^2 = y^2$$

paraboloid.

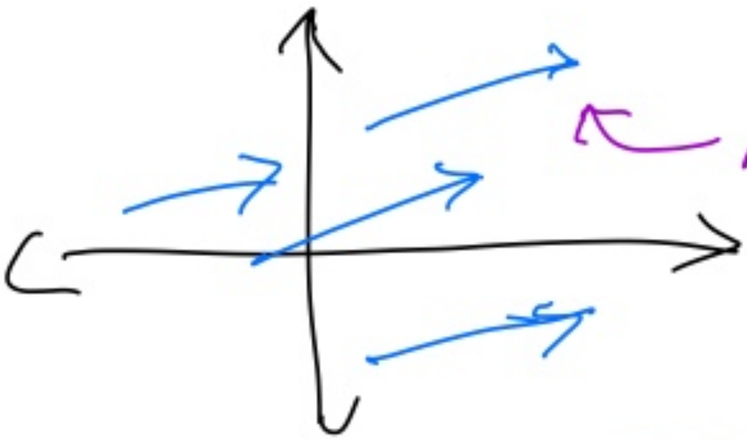
Vectors

$v = B - A$

e.g. vector connecting $\overset{\text{start}}{(2,7)}$ to $\overset{\text{end}}{(-5,4)}$

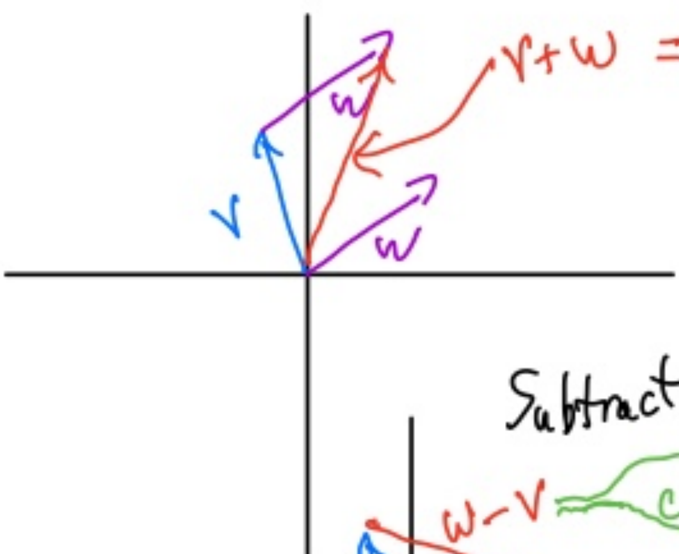
is $(-5-2,\ 4-7) = (-7,-3)$

$= -7\hat{i} - 3\hat{j} = \langle -7,-3 \rangle$.



all considered the same vector as long as the point in the same direction & have the same length.

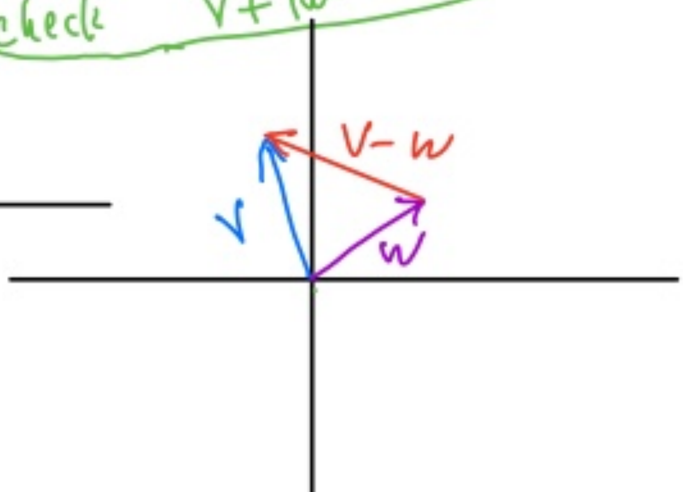
$\Rightarrow$ gives us geometric pictures of addition & scalar mult in $\mathbb{R}^n$.



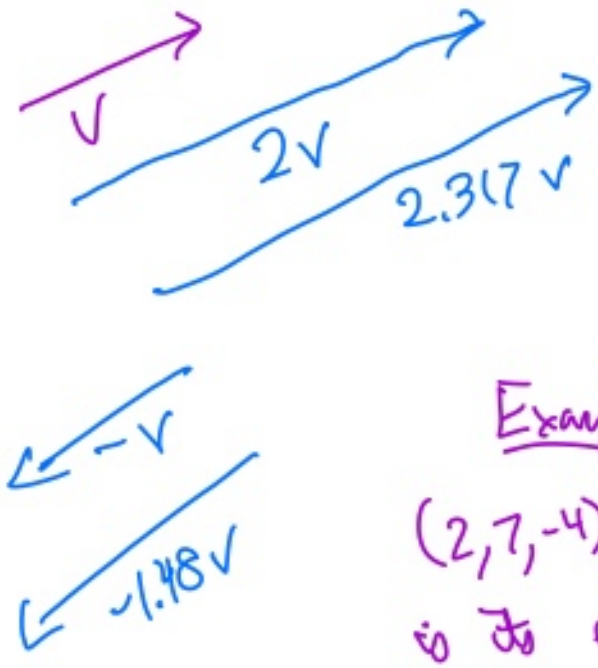
$v + w$ = "resultant" = sum.

Subtraction

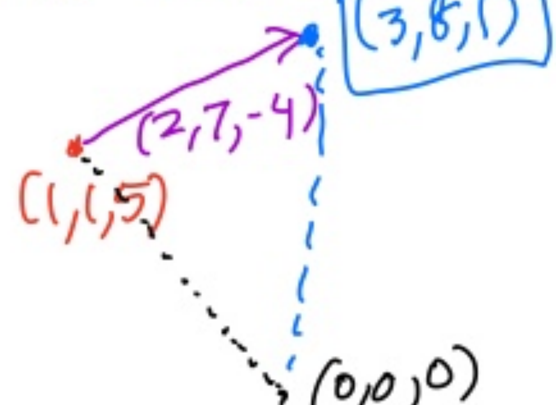
check $v + (w - v) = w$



Scalar mult.



<u>Example</u> If the vector $(2,7,-4)$ starts at $(1,1,5)$, what is its end pt?



Properties of addition & scalar mult.

① $\forall v \in \mathbb{R}^n, w \in \mathbb{R}^n, \; v+w = w+v$ (commutative property of vector +)

② $\forall v, w, z \in \mathbb{R}^n, \; v+(w+z) = (v+w)+z$ (associative property of vector +)

③ $\forall a, b \in \mathbb{R}, v \in \mathbb{R}^n$

$a(bv) = (ab)v$ (associative property of scalar mult.)

④ $\forall a, b \in \mathbb{R},\ v, w \in \mathbb{R}^n$

$$\left.\begin{aligned} (a+b)v &= av + bv \\ a(v+w) &= av + aw \end{aligned}\right\}$$

Distributive properties in $\mathbb{R}^n$.